Modular invariants and NIM-reps

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October 3 2003 – October 18 2003

Most of our time during this program was directed at extending the known results on D-brane charges of WZW models in string theory. Let us begin by sketching the context.

String theory contains two sorts of strings: open strings (*i.e.* strings with two end-points), and closed strings (*i.e.* strings that have the topology of a circle). These strings propagate in some background space (such as for example Minkowski space). The end-points of open strings lie on in general multi-dimensional hupersurfaces that are called D-branes. D-branes are dynamical structures in their own right, and much of their behaviour is captured by their charges. At least for certain examples, the charges $q_a \in Z$ associated to a brane labelled by a, obey an identity of the form

$$D_a\, q_b \equiv \sum N_{ab}^c\, q_c \qquad (\mathrm{mod}\ M)\,, \tag{1}$$

where D_a and M are integers, and the coefficients N_{ab}^c are the so-called fusion coefficients (or more generally, NIM-reps). Understanding D-brane charges is a natural and fundamental problem in string theory.

One of the best-understood string theories are the Wess-Zumino-Witten (WZW) models, for which the background space is a Lie group manifold. The algebraic structure governing the WZW models are affine Kac-Moody algebras, and so these models typically involve very pretty mathematics. Our working hypothesis has been that any natural question asked of a WZW model, should have a Lie theoretic answer. For example, the possible D-branes preserving the full affine algebra symmetry are parametrised by the integrable highest weights of a given level.

What has been worked out already [1, 2, 3] are the charges of D-branes living on a simply connected Lie group manifold $\widetilde{G}$, and preserving the full affine symmetry. They found that the charges q_λ equaled the Weyl dimension of the associated finite-dimensional representation $\overline{\lambda}$, and that M was a certain factor of the level k plus the dual Coxeter number $h^\vee$.

One of the two main questions we addressed, was to extend this to non-simply connected Lie groups G. Each such G corresponds to some subgroup Z_0 of the centre Z of the simply connected cover $\widehat{G}$. This particular question has not been seriously addressed in the literature, partly because it is difficult to find good expressions for the corresponding NIM-rep coefficients N_{ab}^c. We obtained the charges q_a whenever the order of the subgroup Z_0 is a prime. We also showed with concrete examples that the situation for composite $|Z_0|$ is similar though technically more difficult. For primes $p \neq 2$, the answer is reminiscent of that for simply connected groups; however when $|Z_0| = 2$ (*e.g.* $G = SO(3)$) there is an obstruction which unexpectedly trivialises the possible charges. We are currently in the process of typing up this paper [4] and expect to submit it to the preprint server hep-th and the journal JHEP in the very near future.

There are also D-branes which preserve the affine algebra only up to some twist, *i.e.* up to a symmetry of the corresponding Dynkin diagrams. We computed the relevant NIM-rep coefficients in a previous paper [5]. At BIRS [6] we worked out the associated charges, and found them to be dimensions of representations of the corresponding invariant algebra. We also found that the twisted M equaled the untwisted M of [1]. The twisted M had been previously calculated by Braun [7] using completely independent means (K-theory) but the charges were not known.

Fitting this into a broader perspective, the question of D-brane charges can be asked for every possible toroidal partition function. We have a good understanding now of these partition functions for the WZW models: apart from certain small exceptional levels, they correspond to a choice of (not necessarily simply connected) Lie group, and a choice of Dynkin diagram automorphism. The naturality of our new papers [4] and [6] becomes obvious from that perspective. The only generic WZW D-brane charge question which has yet to be addressed, then, corresponds to the twists of the models on non-simply connected groups.

Our two weeks at BIRS were two of our most productive ever. We found the environment beautifully conducive to research. We also found Andrea Lundquist very helpful. We certainly look forward to our next visit!

References

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