

## **NONLINEAR CONSERVATION LAWS AND RELATED PROBLEMS**

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### **Overview of the Field**

Nonlinear Conservation Laws result from the balance laws of continuum physics and govern a broad spectrum of physical phenomena in compressible fluid dynamics, nonlinear materials science, particle physics, semiconductors, combustion, multi-phase flows, astrophysics, and other applied areas. They have a close connection with various geometric problems including low codimension isometric embeddings and the Nirenberg problem of embedding of Riemannian manifolds with prescribed Gauss curvature.

Typical examples of “nonlinear conservation laws” are the Euler equations, MHD equations, Navier-Stokes equations, Boltzmann equation, and other important models arising in Elasticity, Fluid Dynamics, Combustion, and Kinetic theory. The Euler equations for inviscid compressible fluid flow are fundamental and important to many applications, yet the multidimensional theory is difficult and challenging.

In recent years, major progress has been made in both the theoretical and numerical aspects of the field. Motivated by recent results that are bound to revolutionize the field as well as major open problems with great relevance to applications, we organized a 5-day workshop at Banff bringing together experts in the theoretical and numerical aspects of hyperbolic conservation laws and related partial differential equations to take part

in the examination of emerging problems, exchanging ideas in a structured and focused environment. Furthermore, the workshop offered an opportunity to bring into focus on other problems that are able to be addressed by the methods developed by the conservation laws community.

## **Recent Developments and Open Problems**

The theme of the workshop covered several aspects of the theory of weak solutions for hyperbolic systems, the mathematical theory of transport equations that arise in the kinetic theory of gases, and the investigation of the multidimensional Euler, relativistic Euler, Euler-Poisson, Navier-Stokes equations, and related applications of nonlinear conservation laws to physical and geometric problems.

In the area of multidimensional hyperbolic systems, we discussed topics of research in three basic areas, all unified by their showed common dependence on mixed hyperbolic-elliptic equations and geometry:

- (1) unsteady transonic flow and shock reflection-diffraction,
- (2) steady transonic flow and differential geometry,
- (3) mixed systems arising in blood flow modeling.

This research is related to problems in two or more space dimensions. We feel that the subject of conservation laws in one space dimension is relatively mature and understood. It is the multidimensional (and in fact mixed hyperbolic-elliptic) area where major effort is needed.

In the area of Navier-Stokes equations in several space dimensions, existing results and major open problems were discussed on questions of existence and regularity of solutions.

Further presentations on models of compressible fluids were given with focus on applications.

## **Presentation Highlights**

In this section we present a description of the keynote lectures of the meeting as well as the topics discussed during the panel discussions. The main themes of the keynote lectures are:

- (a) Singular Limits in Hydrodynamics
- (b) Recent Developments in Numerical Methods for Nonlinear PDEs
- (c) Local Isometric Embedding of Surfaces with Nonpositive Gaussian Curvature
- (d) Kaehler Geometry from PDE Perspective.

Title: Kaehler Geometry from PDE Perspective

Speaker: Xiuxong Chen, University of Wisconsin-Madison, USA

Abstract: I will explain a few problems naturally arised from Kaheler geometry setting, including the Kaehler Ricci flow, the Geodesic problem and the Kaehler Einstein problems.

Title: On the Dimension of the Navier-Stokes Singular Set

Speaker: Walter Craig, McMaster University, Canada

Abstract: A new estimate on weak solutions of the Navier-Stokes equations in three dimensions gives some information about the partial regularity of solutions. In particular, if energy concentration takes place, the dimension of the microlocal singular set cannot be too small. This estimate has a dynamical system proof.

These results are joint work with M. Arnold and A. Biryuk.

Title: Singular Limits in Thermodynamics of Viscous Fluids

Speaker: Eduard Feireisl, Academy of Sciences, Czech Republic

Abstract: We discuss some recent results concerning singular limits of the full Navier-Stokes-Fourier system, where one or several characteristic numbers become small or tend to infinity.

The main ingredients of our approach read as follows:

- (i) A general existence theory of global-in-time weak solutions for any finite energy data;
- (ii) Uniform bounds independent of the value of the singular parameters based on total dissipation balance (Second Law of Thermodynamics);
- (iii) Analysis of propagation of acoustic waves, in particular on unbounded or ``large' spatial domains.

Title: Recent Developments in Conservation Laws

Speaker: Hermano Frid, Instituto de Matemática Pura e Aplicada  
IMPA, Brazil

Abstract: The following topics will be presented:

- (i) Homogenization of nonlinear PDE's, including nonlinear transport equations, porous medium type equations, etc., in ergodic algebras, by using two-scale Young measures.
- (ii) Global existence of spatially periodic solutions for certain 3X3 hyperbolic-parabolic systems in gas dynamics.
- (iii) Well-posedness of initial-boundary value problems for quasilinear multi-dimensional parabolic systems in multiphase flows in porous media and filtration.

Title: Local Isometric Embedding of Surfaces with Nonpositive Gaussian Curvature

Speaker: Qing Han, University of Notre Dame, USA

Abstract. In this talk, we introduce a new linearization process for the Darboux equation, a fully nonlinear differential equation for the isometric embedding of 2-dimensional Riemannian manifolds in  $\mathbb{R}^3$ . We prove that the local solvability of the Darboux equation is determined solely by the local solvability of this linearized equation. In 1873, Schlaefli conjectured that every 2-dimensional smooth Riemannian manifold admits a smooth local isometric embedding in  $\mathbb{R}^3$ . It was more than 50 years later that an affirmative answer was given for the analytic case by Janet; he proved in 1926 that any 2-dimensional analytic Riemannian manifold admits a local analytic isometric embedding in  $\mathbb{R}^3$ . Schlaefli's conjecture for the smooth case was given a renewed attention by Yau in the 1980s and 1990s. The existence of a local isometric embedding of a 2-dimensional smooth Riemannian manifold in  $\mathbb{R}^3$  can be shown easily to be equivalent to the existence of a local solution of a fully nonlinear equation of the Monge-Ampere type.

Title: Global Semigroup of Solutions of the Nonlinear Variational Wave Equation

Speaker: Helge Holden, Norwegian University of Science and Technology, Trondheim, Norway

Abstract: We prove the existence of a global semigroup for conservative solutions of the nonlinear variational wave equation  $u_{tt} - c(u)(c(u)u_x)_x = 0$ . The equation exhibits focusing of energy, and the behavior of solutions is rather subtle. The analysis is performed by a careful choice of new variables. A conservative numerical method is presented as well.

Title: Discrete Duality Finite Volume Schemes for Doubly Nonlinear Degenerate Hyperbolic-Parabolic Equations

Speaker: Kenneth H. Karlsen, University of Oslo, Norway

Abstract: We consider a class of doubly nonlinear degenerate hyperbolic-parabolic equations with homogeneous Dirichlet boundary conditions, for which we first establish the existence and uniqueness of entropy solutions. We then turn to the construction and analysis of discrete duality finite volume schemes (in the spirit of Domelevo and Omnes) for these problems in two and three spatial dimensions. We derive a series of discrete duality formulas and entropy dissipation inequalities for the schemes. We establish the existence of solutions to the discrete problems, and prove that sequences of approximate solutions generated by the discrete duality finite volume schemes converge strongly to the entropy solution of the continuous problem. The proof revolves around basic a priori estimates, the discrete duality features, Minty-Browder type arguments, and "hyperbolic"  $L^\infty$  weak- $\star$  compactness arguments (i.e., propagation of compactness along the lines of Tartar, DiPerna, ...). Our results cover the case of non-Lipschitz nonlinearities. This is joint work with B. Andreianov and M. Bendahmane.

Title: Some Recent Research Projects on Hyperbolic Problems

Speaker: Randy LeVeque, University of Washington, USA

Abstract: Finite volume methods on the sphere using logically rectangular quadrilateral grids that do not suffer the "pole problem" of

latitude-longitude grids. Adaptive mesh refinement on these grids. Applications to shallow water equations on a rotating sphere over topography and well-balanced methods for accurately modeling small amplitude waves. (Joint work with Marsha Berger, Donna Calhoun, and Christiane Helzel).

Geophysical flow problems, including tsunami modeling, storm surges, debris flows, and submarine landslides using depth-averaged models (extensions of shallow water equations). Issues include well-balancing for source terms, dry-state Riemann solvers at the margins of the flow, loss of hyperbolicity in layered shallow water equations, nonconservative products arising in some models. Development of GeoClaw software for solving such problems with adaptive mesh refinement. (Joint work with David George, Marsha Berger, Kyle Mandli, Jihwan Kim, and Roger Denlinger).

Shock wave propagation in biological media related to shock wave lithotripsy and therapy. Elastic wave propagation and reflection at tissue/bone interfaces. (Joint work with Kirsten Fagnan, and Tom Matula).

Extensive rewriting of the Clawpack software to improve user interface, incorporate Python graphics and tools for reproducible research (Joint work with Kyle Mandli, Chris Swierczewski).

Title: Dissipative Systems

Speaker: Tai-Ping Liu, Academia Sinica (Taipei) and  
Stanford University, USA

Abstract: We will survey three types of dissipative systems: the hyperbolic conservation laws, the viscous conservation laws, and the Boltzmann equation. The concept of entropy runs through all these systems. Analytically, they are closely related, and yet major differences exist. We will discuss these and raise open questions.

Title: Weak Solutions of the 2D Pressureless Equations  
Speaker: Eitan Tadmor, University of Maryland, USA  
Abstract: We propose a new framework to study weak solutions of the two-dimensional pressure-less equations. Solutions are constructed by vanishing viscosity method, using new uniform BV estimates. The (strong) viscosity limits are interpreted as weak solutions of the pressure-less equations using duality arguments. (A joint work with Dongming Wei).

Title: A Kinetic Model for the Sedimentation of Rod-Like Molecules  
Speaker: Athanasios Tzavaras, University of Maryland, USA  
Abstract: The sedimentation of suspensions of rod-like particles shows interesting pattern formations that have been studied by several authors in theoretical, numerical and experimental work. Here we consider a kinetic multi-scale model which couples a microscopic Smoluchowski equation (for the distribution of rod orientation) to a macroscopic Stokes or Navier-Stokes equation with an elastic stress tensor and a gravitational forcing term. A reciprocal coupling of phenomena on these different scales leads to the formation of clusters. We analyze these models in various scaling regimes to describe the aggregate behavior. In a diffusive scaling limit the aggregate behavior is described by the Keller-Segel model.

Title: On Transonic Flows in Nozzles  
Speaker: Zhouping Xin, Chinese University of Hong Kong  
Abstract: In this talk, I will discuss some progress on studying steady compressible flows in a class of general nozzles, in particular, on the well-posedness of transonic shocks in a multidimensional de Naval nozzle, existence of subsonic-sonic flows in general multidimensional nozzles.

### **Outcome of the Meeting**

Here are some remarks. First we came to realize the key role of mixed

hyperbolic-elliptic systems of partial differential equations. This was no surprise since these problems arise naturally in both shock reflection-diffraction and transonic flow, as well as isometric embedding. Hence we plan to create a research program emphasizing mixed problems both in their classical role in compressible fluid flow and in their applications in bio-mechanics and differential geometry. We think that this is an area ripe for breakthroughs on questions that had often been left in research monographs in the category of “unsolved open problems” (e.g., Bers, page 135; Courant-Friedrichs, page 317; Lax, page 427; Morawetz, page 24; Yau, page 355). The physical geometry of our fluid problems seems to be playing a major role in our analysis. For example, special small scale effects of airfoil shape seem to be playing a crucial role.

### **Program of the Meeting: October 4-9, 2009**

October 5, Monday

9:00-10:00: Survey Talk: Randy Leveque

10:05-10:45: Y. Ha

Break

11:00-11:40: Ronghua Pan

11:45-12:25: Tzavaras

Lunch+discussion

3:00-3:40: Tadmor

3:45-4:25: Klingenberg

Break

4:45-5:25: Robert McCann

October 6, Tuesday:

8:30-9:30: Survey Talk: Eduard Feireisl 9:35-10:15: Fabio Ancona

Break

10:40-11:20: Donadello Carlotta

11:25-12:05: Marcati

Lunch+Discussion

2:00-2:40: Xin

2:45-3:25: Spinolo Break

4:00-4:40: Hermano Frid

October 7, Wednesday:

8:30-9:30: Survey Talk: Qing Han  
9:35-10:15: Yongqian Zhang  
Break  
10:40-11:20: Walter Craig  
11:25-12:05: Kenneth Karlsten

Lunch+Discussion  
Free Afternoon

8:00-9:30pm: Panel Discussion  
Chair: Constantine Dafermos

October 8, Thursday:  
8:30-9:30: Survey Talk: Xiuxiong  
9:35-10:15: D. Serre  
Break  
10:45-11:25: Tai Ping Liu  
11:30-12:10: M. Torres

Lunch+discussion  
2:00-2:40: Helge Holden  
2:45-3:25: Zhu Yi  
Break  
4:10-4:50: M. Feldman

October 9, Friday:  
8:30-9:10: Gao Shu  
9:15-9:55: Dianwen, Zhu  
10-10:40: Bae, Myoungjean  
Free Discussion

11:30: End of Workshop

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