THREE THEORETICAL PROBLEMS IN THE CONTROL OF ROTATING MACHINES

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1 Introduction

The timing of this focussed research group (FRG) was excellent. Five of the six participants (from Canada, Spain, and the UK) are all involved in a 3-year research programme funded by the UK Engineering and Physical Sciences Research Council (EPSRC) for the three year period, July 1, 2007 to June 30, 2010. Thus, the FRG provided a golden opportunity to reflect on achievements under this aegis, and to consider future activities of a similar kind.

We consider the EPSRC programme to have been highly successful. Bringing together personnel with diverse backgrounds in engineering, mathematics, and computation has been productive, and reveals some exciting vistas for continued collaboration.

The daily programme consisted of a collective working session in the morning, relaxation in the early afternoon, collective and/or private working sessions later in the day.

2 Overview of the Field

There is a class of mathematical models of linear systems which occurs in a geat variety of physical and engineering problems; namely second order differential systems

$$M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = f(t).$$

Depending on the context, the setting may be in infinite or finite dimensional spaces, and analysis frequently leads to study of time-independent nonlinear eigenvalue problems of the form

$$(\lambda^2 M + \lambda D + K)p = g \tag{1}$$

where λ is a real or complex parameter. Again, the context may be either infinite or finite-dimensional vector spaces. In particular, when it comes to computation it is *finite-dimensional* models which must be understood (when M, D, K are almost always square matrices - possibly very large). This is the mathematical environment for this FRG. It is also the environment for a vast body of literature in engineering and applied mathematics. Spectral theory, stability, control and optimization, and the design of algorithms are among the major concerns.

In many problems M, D, K enjoy symmetries. For example, for physical reasons, M is frequently positive definite, and similarly for D and K, but general purpose algorithms cannot assume that this will always be the case. Also, in the analysis of rotating machinery, D, and possibly K, will have a significant skew-symmetric part and, if a mechanical system is free to move in at least one space dimension, M may be singular.

If the model is infinite-dimensional it is necessary that, for the purpose of computation, finite dimensional approximations be made. We do not consider the associated "truncation" errors, but work in the context of the resulting finite-dimensional systems, $\lambda^2 M + \lambda D + K$. Much of the activity in analysis and computation involves the formulation of systems which are isospectral with with $\lambda^2 M + \lambda D + K$, but are of *first* degree in the parameter λ. This is a process known as "linearization". Efficient algorithms for eigenvalues act on the linearized system and will often preserve their characteristic "block" structure - leading to the idea of "structure-preserving transformations" which are also isospectral (i.e. preserving the eigenvalues, their multiplicities and, in the case of real eigenvalues, their "sign characteristic").

In general, there is an insatiable appetite "out there" for faster, more robust algorithms tailored to particular problem areas, and capable of handling larger and larger problem sizes. This provides constant stimulation in the study of algorithms.

3 Recent Developments and Open Problems

In engineering practice there is great dependency on models in which the three matrix coefficients M, D, K can be diagonalized simultaneously; and the so-called "modal analysis" works in this context. This has been carefully examined in [7] and expressed in mathematical terms - and provides a useful basis for some of our subsequent investigations.

Research of the group has focussed primarily on the theory and practice of diagonalizing $L(\lambda) = M\lambda^2 + D\lambda + K$ by the application of simple λ-dependent transformations which we call "filters". This can be done without recourse to linearization strategies. Thus, we show that (in general), if systems $L(\lambda) = M\lambda^2 + D\lambda + K$ and $\tilde{L}(\lambda) = \tilde{M}\lambda^2 + \tilde{D}\lambda + \tilde{K}$ are isospectral, then there exist *first degree* polynomials $\tilde{F}(\lambda)$ and $F(\lambda)$ such that

$$\tilde{F}(\lambda)L(\lambda) = \tilde{L}(\lambda)F(\lambda).$$

The "decoupling" then consists in finding filters for which $\tilde{L}(\lambda)$ is *diagonal*. The importance of this is that there are mechanisms admitting physical implementation of such filters using *feedback* mechanisms (familiar in engineering and control theory).

Numerical experiments of S.D.Garvey with S.Jiffri suggest that there are constraints on the spectra of achievable filters which require further analysis. This is in progress and is complemented by the physical construction of model systems (see below). In particular, it has been discovered that filters can be expressed in terms of the "structure preserving transformations" mentioned above, and this is the central idea of [2]. (It has also been found that structure preserving transformations can play a role in perturbation theory [1]).

In the context of linearized systems a thorough analysis has been made of the nature of (suitably defined) structure preserving transformations. Complete parametrizations of such transformations are obtained in [8] in terms of the centralizer of the underlying Jordan structures. This includes systems with or without symmetries in the coefficients M, D, K.

In this context, discussions at the FRG have revealed some gaps in our understanding of canonical "Jordan canonical triples" for real symmetric quadratic systems. This involves some fundamental algebraic ideas and has stimulated refinements in the paper [9] - now near completion.

4 Presentation Highlights

S. D. Garvey made a presentation concerning the Garvey/Popov/ Lancaster project on the application of a Rayleigh-quotient algorithm to "dilated" quadratic systems. This admits calculation in real arithmetic for real or complex eigenvalues and treats defective eigenvalues seamlessly. This presentation stimulated further polishing of the paper [3].

I. Zaballa presented a careful analysis of a new subspace iteration technique for triangularizing quadratic matrix polynomials.

A. A. Popov described the development of test-rigs nearing completion at the University of Nottingham to be used in collecting sample data to test our theories and algorithms. There are two such experiments in progress. One is made up of electrical circuits with controllable voltage, and the other is a mechanical rotor with controllable bearings.

F. Tisseur made a presentation concerning work of her group on *in situ* “deflation” of quadratic systems (without recourse to linearization). Her collaborations with members of the group (see [4] and [6]) suggest her inclusion in future plans (so providing expertise in computational science).

5 Scientific Progress Made

As mentioned above, fundamental properties have been found pertaining to Jordan canonical forms - in the context of hermitian and, particularly, real symmetric systems. In the FRG meeting, and at the time of writing, these are still being polished. They give new and important insights into spectral structures. In particular, we obtain fundamental orthogonality properties of eigenvectors providing natural generalizations of classical properties of real symmetric matrices. This points the way to extensions of the Lancaster/Prells studies of orthogonality properties of eigenvector structures (see [5]).

More generally, the FRG provided a golden opportunity for assessing achievements to date and for planning future research directions.

6 Outcome of the Meeting

This FRG meeting allowed the participants to assess their three years of group-research, to polish and re-evaluate investigations nearing completion and, most important, to look ahead, identify problem areas, and consider future plans. These will include further funding applications in the UK, extending the Canadian participation and, possibly, seeking Canadian funding. It was also agreed that the directions of research would include more specialist expertise in computation.

References

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