Combinatorial and Geometric Discrepancy

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September 30 – October 2, 2020

1 General Information

The workshop on Combinatorial and Geometric Discrepancy was intended as a one-week workshop held at BIRS. Because of the international covid crisis, the workshop could not be held as originally planned, and will hopefully take place as an in-presence workshop at BIRS in 2021 or 2022 instead. We decided against holding the full workshop as an online event; however, to give some of the young (potential) participants an opportunity to present themselves and their research work, we organized two half-day session with online talks. All the talks were held by young researchers (typically early-stage PostDocs). Additionally, there was an open problem and discussion session.

2 Overview of the Field

Discrepancy theory deals with problems concerning the existence and the construction of configurations exhibiting a very high degree of regularity, usually tested with respect to a supremum norm or other norm over a large class of test objects. A classical situation are finite point sets in the d-dimensional unit cube $[0,1]^d$, with the class of test sets being the class of all axis-parallel boxes contained in $[0,1]^d$: the relative number of elements of the point set in a test box is compared with the volume, and then a supremum over all boxes in the class of test sets is taken. There are also notions of discrepancy which are of a much more combinatorial flavor, such as for example red-blue colorings of points sets. Many interesting questions concerning the smallest possible value of the discrepancy in a particular setup remain unsolved, but it is a difficult and important challenge to find (algorithmically efficient) constructions of low-discrepancy sets. Discrepancy theory is closely connected with many mathematical areas, including harmonic analysis, number theory, numerical analysis, ergodic theory, combinatorics, to name just a few. The main idea of our workshop was to bring together researchers from different mathematical areas, who are linked by their common interest in discrepancy-related topics, but who often are not closely following the developments and the methods used by other researchers working on related problems but using the language of another mathematical discipline.

3 Presentations

As noted above, the main idea of our online mini-workshop was to give young researchers a possibility to introduce themselves and their research area to other members of the discrepancy theory community. In total

we had 9 speakers, and around 75 persons in the audience. The duration of the talks was 25 minutes each, and the speakers were asked to give non-technical and accessible talks, considering the diverse mathematical background of the members of the audience. From members of the audience we heard that the event was very entertaining and pleasant to follow. At the end of the workshop we held an open problem and discussion session, which was well attended and which some young researchers used (in the spirit of the event) to indicate that their working contracts were about to expire and that they were looking for a new (PostDoc) position.

The following talks were given during the online-workshop:

- Ryan Alweiss (Princeton University): Discrepancy Minimization via a Self-Balancing Walk
- Samantha Fairchild (University of Washington): Families of Well-approximable Measures
- Sebastian Neumayer (TU Berlin): Curve Based Approximation of Images on Manifolds
- Tetiana Stepaniuk (Universität Lübeck): Hyperuniformity of Point Set Sequences
- Hendrik Pasing (Hochschule Ruhr West): Improved Discrepancy Bounds and Estimates
- Ujue Etayo (TU Graz): A Deterministic Set of Spherical Points with Small Discrepancy
- Mathias Sonnleitner (JKU Linz): (Non-)optimal Point Sets for Numerical Integration
- Victor Reis (University of Washington): Vector Balancing in Lebesgue Spaces
- Lily Li (University of Toronto): On the Computational Complexity of Linear Discrepancy