Symmetries of Gravity at the Black Hole Horizon

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1 Overview of the Field, Recent Developments, and Open Problems

The last decade has been a thriving period in theoretical and mathematical physics. Old results, such as Noether theorems [1] or the covariant phase space formalism [2, 3, 4, 5, 6] have received new inputs, that helped us both in revitalizing the field and refining these important results. The progress in understanding asymptotically flat holography, Carrollian physics, black hole horizons, and the information paradox all have the concept of symmetry as the common denominator.

Specifically, gauge symmetries play a pivotal role in all these fields. Whenever gauge symmetries are pulled back to boundaries of interest, part of these symmetries carries non-vanishing conserved quantities. These are instrumental in organizing observables of the theory. A deeper appreciation of these features of gauge theories led to the formulation of the corner proposal [7, 8, 9]. The latter is based on the fact that gauge symmetries give rise to codimension-2 conserved quantities, called therefore corner charges – where the term 'corner' indicates a generic such codimension-2 surface. Then, the corner proposal states that, in the context of gravitational theories, these corner charges and their algebra should be considered as a guiding principle toward quantum gravity; see [10] for more details, and a complete list of references on the topic.

One of the main obstacles that the corner proposal overcame is that on a typical boundary there are fluxes, and thus we are in the presence of an open system. It has been shown [11, 12] that one can still consider canonical charges, if one extends the phase space to include the embedding of the corner as a dynamical map. This allows us to consider Poisson brackets even in the presence of leakage, and thus opens the door to a canonical quantization of open systems, something that has never been possible before.

Consider gravity as our gauge theory of interest. Then, among possible boundaries of interest, the black hole horizon plays a prominent role. Indeed, we have found in recent years the appearance of enhanced asymptotic symmetries on this boundary [13, 14]. Recasting these results in the framework of the corner proposal is one first open question. While so far these symmetries have been considered from a bulk-to-boundary perspective, the corner proposal gives a radically different viewpoint, and allows us to wonder whether from a single corner at the horizon we can locally reconstruct the two normal directions. Furthermore, recasting these near horizon results on the extended phase space aforementioned, we can propose a Poisson bracket among canonically realized charges at the horizon, even when fluxes leak through this boundary.

Another reason why the black hole horizon is such an important boundary is that it has been a source of unresolved paradoxes. The most notable one is the information paradox, based on Hawking's old observation [18] that there would be loss of predictability of the final state if the black hole evaporated completely, because one could not measure the quantum state of what fell into the black hole. How does the information of the quantum state of the infalling particles re-emerge in the outgoing radiation? Hawking, Perry and Strominger [15, 16, 17] proposed in recent years a resolution of this paradox based on the symmetries of gravity at the black hole horizon. More technically, they proposed that the information is stored in a specific BMS transformation (a supertranslation, called soft hair) associated with the shift of the horizon caused by ingoing particles, and they argued that such effects can still be measured after the black hole evaporates, leading to no loss of information in the process. Their proposal is important in that it focuses on the subject on symmetries and conserved quantities. Nonetheless, it is still an open problem to mathematically formulate their resolution on the horizon, in particular the rigorous construction of a phase space underlying their proposal is still missing at present. A full understanding of this conundrum requires to study the time scale at which the semiclassical approximation holds no more, known as the Page time. Following the evaporation process with charge evolution and fluxes throughout is still an open problem, and we believe it to be accessible in the framework of the corner proposal.

2 Scientific Progress Made

We followed three avenues of investigations. The first one concerns the asymptotic symmetries at the horizon. By recasting previous works in a gauge-independent approach, and enlarging the phase space, we found a framework where flux (indicating matter falling into the horizon) are included in the phase space, and are accounted for by the variation of the embeddings. The Noetherian split between charges and flux proposed in [9] differs from the original split used in [13], and allows us to easily include expanding horizons (varying surface gravity) in the phase space. The last step in this first topic was to discuss the moment map, needed to map the symplectic structure at horizons to symplectic leaves of the coalgebra (following the coadjoint orbit method of Kirillov, see e.g. [19] and again [10] for more references). This shows that a preferred polarization of the phase space is selected, and we are still investigating the consequences of this.

The second topic we investigated is the extension from one corner at the horizon to two corners at different times. The corner proposal is based on the idea of local holography. Thanks again to the extended phase space, we managed to evaluate the Poisson bracket between charges at different normal times, exploring the vast yet uncharted territory of spacetime reconstruction. In particular, if the time coordinate separating the two corners is null, this bracket is ultra-local in time. We stress that this was doable thanks to the extension of the phase space, which makes charges canonical even in the presence of fluxes. This gives us a fresh perspective on the topic. Various directions then unfold. One is to explore the reconstruction, while the other is to study how this two corners commutator helps us in addressing the black hole information paradox.

This is the last avenue we pursued, in which we made only very preliminary progress. One of the problems in the common treatment of black hole evaporation is the assumption that we can continue to treat the problem semiclassically even after the Page time, that signals the point in time after which the evaporated Hawking radiation has more entropy than the black hole. Our two corner analysis allows us to compute the Page time purely from asymptotic symmetries, and to show that the evaporated mass is 'piling up' at infinity. This supports a canonical phase space analysis to the Hawking, Perry and Strominger proposal [16], and shows that the theory of gravitational symmetries is still one of the best tools we have to address this conundrum.

3 Outcome of the Meeting

We plan to write a manuscript for each of the avenues mentioned in the previous section. The first paper will be about near horizon symmetries revisited in the framework of the corner proposal and the extended phase space. There, the Noetherian split will be imposed and justified, and the fluxes will be shown to be included as part of the phase space. The important outcome will be the inclusion of the surface gravity and spin-2 fluxes in the phase space, and thus the description of an expanding horizon admitting infalling matter.

In a separate paper we will talk about the two corner computation, and the idea of local holography in the corner proposal will become more concrete. Recasting Einstein's evolution equations and the geometric constraints at the level of charges allows us to appreciate an emerging simple structure, where the spin-0 soft data at corners satisfy commutator relations that, once quantized, gives us a non-commutative geometry.

Further in the future, we plan to report on the information paradox and the computation of the Page time purely from asymptotic symmetries. While this is still at an embryonic stage, the tools and results of the previous two papers will be instructive in defining the path to pursue for this last outcome.

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